

Chapter 9 / **Example 13****Diagonalisation and powers of a matrix**

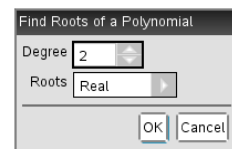
- a** Find the diagonalization of $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$
- b** Hence find an expression for \mathbf{A}^4 in the form $\mathbf{PD}^4\mathbf{P}^{-1}$.
- c** Find an expression for \mathbf{A}^4 as a product of 3 matrices with no exponents.

$$|\mathbf{A} - \lambda \mathbf{I}| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & 2 \\ 3 & -\lambda \end{vmatrix} = 0 \Leftrightarrow \lambda^2 - \lambda - 6 = 0$$

Open a new document and add a Calculator page.

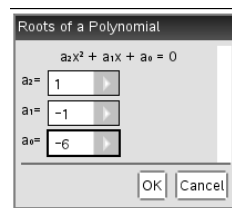
Press **menu** 3:Algebra | 3:Polynomial Tools | 1:Find Roots of Polynomial...

Select degree 2 as this is a quadratic equation.



Enter the coefficients of the quadratic equation.

Press **enter**.

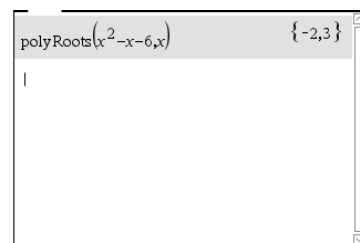


Press **enter**.

The calculator displays the solution $\lambda_1 = 3, \lambda_2 = -2$

The eigenvectors are

$$\mathbf{x}_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \text{ and } \mathbf{x}_2 = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$$

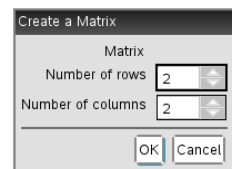


$$\mathbf{P} = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix}$$

Press **menu** 7:Matrix & Vector | 1:Create | 1:Matrix

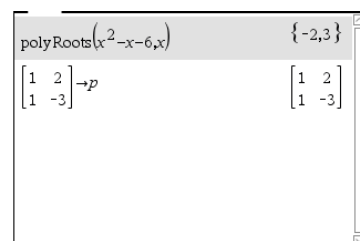
Set the number of rows and columns to 2.

Press **enter**.



Enter the values of the elements of the matrix \mathbf{P} , using **tab** to move through the matrix.

Press **ctrl** **var** **sto→** P and press **enter**.



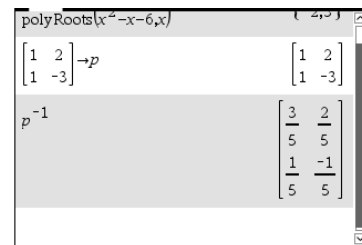
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Type P , press \wedge and type -1 .

Press enter .

The GDC displays the matrix P^{-1}

$$P^{-1} = \begin{pmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$



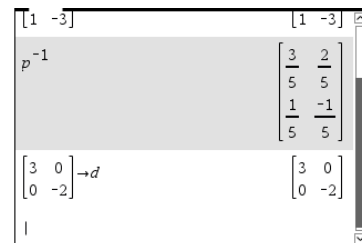
$$D = \begin{pmatrix} 3 & 0 \\ 0 & -2 \end{pmatrix}$$

Press menu 7:Matrix & Vector | 1:Create | 1:Matrix

Set the number of rows and columns to 2.

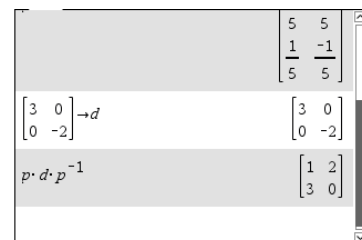
Enter the values of the elements of the matrix D , using tab to move through the matrix.

Press ctrl var $\text{sto} \rightarrow$ D and press enter .



$$A = PDP^{-1}$$

Type $P \times D \times P^{-1}$ and press enter .



$$A^4 = P^{-1}D^4P$$

Type D^4 and press enter .

$$D^4 = \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix}$$

$$A^4 = \begin{pmatrix} 1 & 2 \\ 1 & -3 \end{pmatrix} \begin{pmatrix} 81 & 0 \\ 0 & 16 \end{pmatrix} \begin{pmatrix} \frac{3}{5} & \frac{2}{5} \\ \frac{1}{5} & -\frac{1}{5} \end{pmatrix}$$

